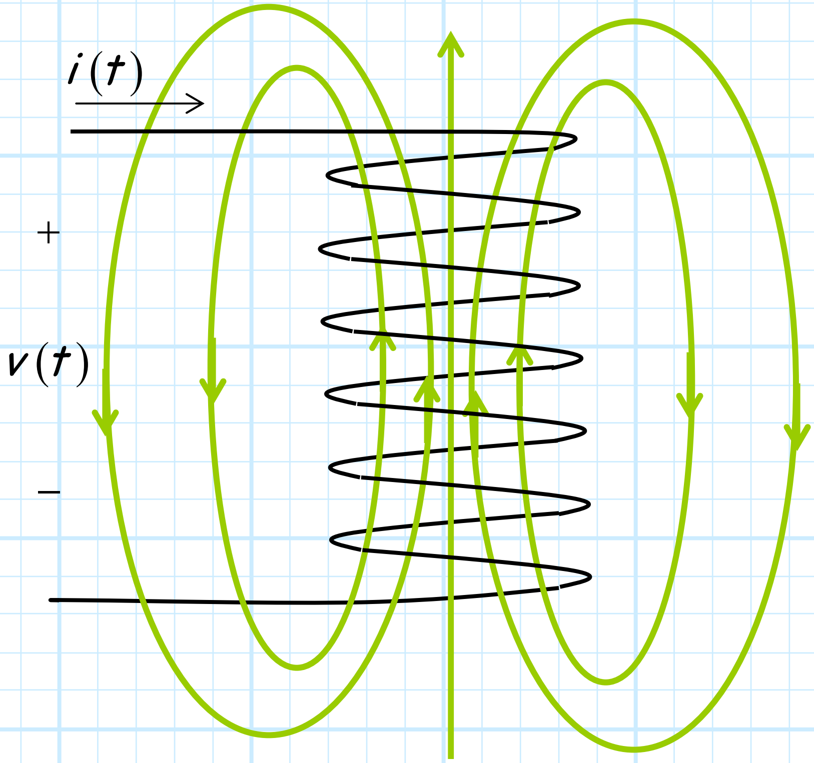
Inductance

Consider a **solenoid** with N turns:



The current $i(t)$ in flowing in the wire will produce a time-varying magnetic flux density within the solenoid. This time-varying magnetic flux density will **induce a voltage** $v(t)$ across the solenoid.

This voltage can be determined using **Faraday's Law**:

$$-\oint_{C_1} \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = \frac{\partial}{\partial t} \iint_{S_1} \mathbf{B}(\bar{r}) \cdot \overline{ds}$$

Just like we determined for the **ideal transformer**, we find that:

$$-\oint_{C_1} \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = v(t)$$

and that:

$$\frac{\partial}{\partial t} \iint_{S_1} \mathbf{B}(\bar{r}) \cdot \overline{ds} = \frac{\partial}{\partial t} N \iint_{S_0} \mathbf{B}(\bar{r}) \cdot \overline{ds}$$

$$= N \frac{\partial \Phi(t)}{\partial t}$$

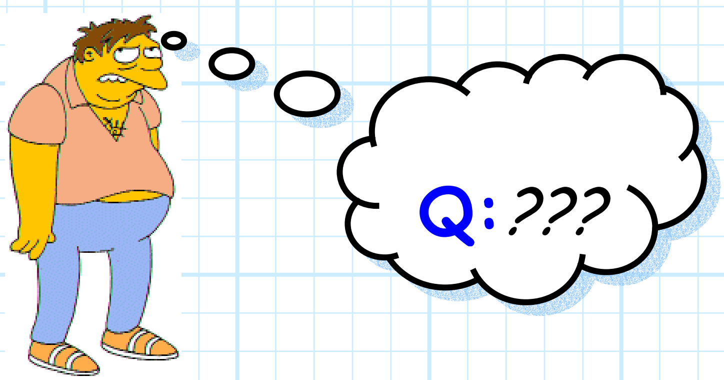
where S_0 is the surface area of **one** loop.

Therefore, just as we determined for a transformer, Faraday's Law says that:

$$v(t) = N \frac{\partial \Phi(t)}{\partial t}$$

Now, let's **define** the product $N\,\Phi(t)$ as:

$$N\,\Phi(t) \doteq \Lambda(t) = \text{flux linkages} \quad [\text{Webers}]$$



A: A magnetic flux of $\Phi(t)$ Webers passes through **each and every** one of the N loops of the solenoid. We say therefore that each loop surrounds, or "**links**" $\Phi(t)$ Webers of flux. If there are N loops, then the solenoid links a **total** of $N\,\Phi(t)$ Webers of flux. We call therefore $N\,\Phi(t)$ the total **flux linkages** surrounded by the solenoid.

Thus we can state our induced solenoid voltage as the time derivative of the flux linked by the solenoid:

$$v(t) = \frac{\partial \Lambda(t)}{\partial t}$$

Now, recall that current *i(t)* produced the magnetic flux density and thus the magnetic flux. As a result, we find that the current *i(t)* is **directly proportional** to the total flux linkages of the solenoid:

$$i(t) \propto \Lambda(t)$$

Lets define the **proportionality constant** as L, so that we can say:

$$\Lambda(t) = L\ i(t)$$

Since *i(t)* has units of amps and $\Lambda(t)$ the units of Webers, the constant L must have units of **Webers/Amp**.

Taking the **time derivative** we thus find:

$$\frac{\partial \Lambda(t)}{\partial t} = L \, \frac{\partial i(t)}{\partial t}$$

Note we can now write the **induced voltage** as:

$$v(t) = L \frac{\partial i(t)}{\partial t}$$

Q: Look familiar?

A: *Of course, L is **inductance**!*

Inductance is therefore defined as the **ratio** of current i to the total flux linkages it creates!

$$L \doteq \frac{\Lambda}{i} = \text{inductance} \quad \left[\frac{\text{Webers}}{\text{Amp}}\right]$$

Inductance is obviously dependent on the **structure** of the device (e.g., number of loops, diameter, length).

By the way, we have another name for Webers/Amp—**Henries**!

$$\text{Henries} \doteq \frac{\text{Webers}}{\text{Ampere}}$$